Lesson 1-6: Coordinate Plane

A fly on the ceiling?

Way back in the early 1600's, there was a guy named René Descartes. Descartes was sick and stuck in bed…bored out of his mind. Lying there, all he had to keep himself occupied was watching a fly buzzing around the ceiling. Watching the fly move around, he realized its position at any instant could be specified by giving its distance from the two walls meeting at the corner. For instance, at one moment the fly may be 2' from the corner along the west wall, and 3' from the corner along the south wall. He then realized that he could write down an equation that gives one of the two distances in terms of the other…as the fly buzzed around the ceiling, he could describe its path algebraically.

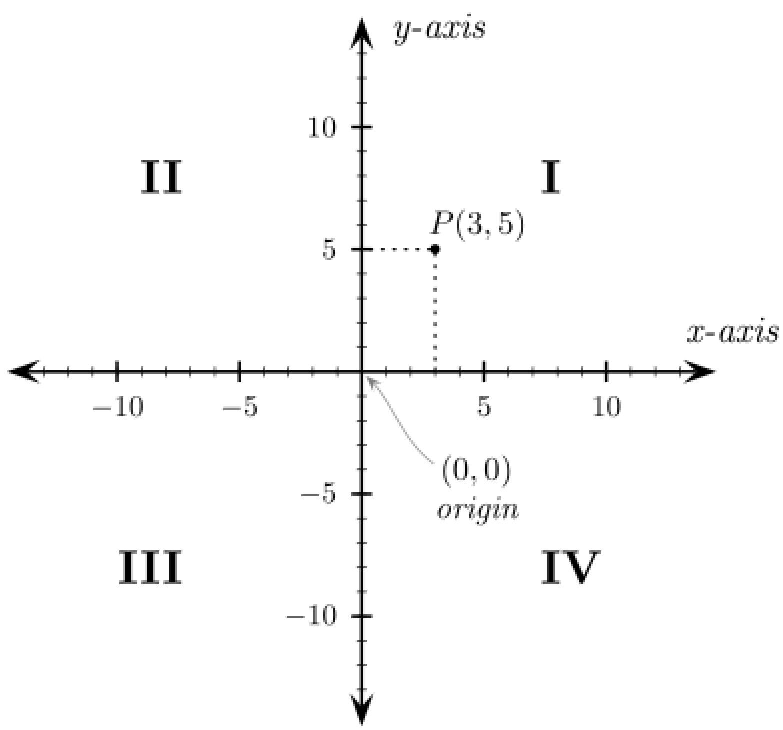
And so the Cartesian coordinate system was born.

(*The Language of Mathematics*, Kevin Delvin, 2002, p. 156)

The Cartesian coordinate system reviewed

Descartes basically took two numbers lines, one horizontal and the other vertical, and made his coordinate plane. The horizontal number line is called the *x*-axis; the vertical is called the *y*-axis. The point at which they cross is the *origin*. Values on the x-axis are positive and increase to the right of the origin, negative and decreasing to the left. Values on the *y*-axis are positive and increase above the origin, negative and decreasing below.

Points are plotted and referenced on the coordinate plane by the *x*-axis value first, *y*-axis value second. This gives us an ordered pair (*x*, *y*).



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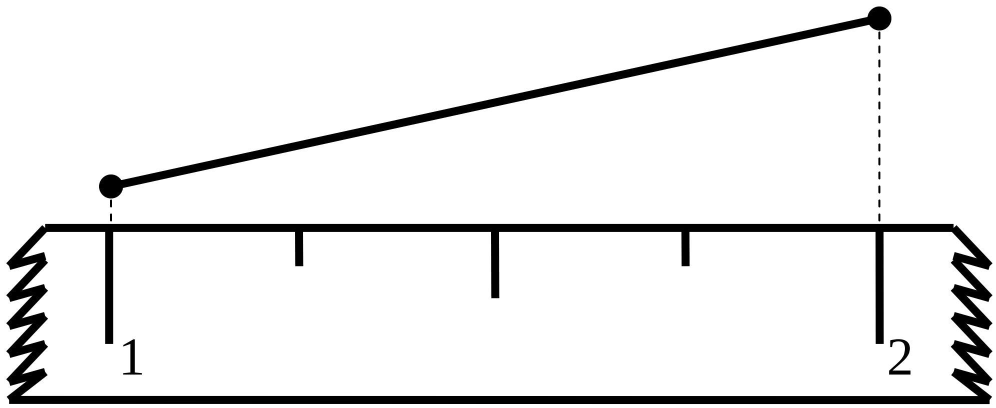
The ruler postulate expanded

Do you recall what the ruler postulate tells us? It allows us to find the distance between any two points on a line, by "projecting" the line onto a ruler. Now this works great as long is the line is parallel with the ruler. But what if the line is slanted?



The line segment above "projects" onto the ruler with a smaller footprint than the actual segment. It isn't an accurate measurement.

This is where Descartes' coordinate system comes in along with a formula that may be familiar.

The distance formula

The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We are going to explore the geometric origin and meaning of this formula deeper in chapter 7 later.

Example (pg. 46, #6)

E(6, -2) $x_1 = 6$, $y_1 = -2$

F(-2, 4) $x_2 = -2$, $y_2 = 4$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-6)^2 + (4-(-2))^2} \\ &= \sqrt{(-2-6)^2 + (4+2)^2} \\ &= \sqrt{(-8)^2 + (6)^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

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Midpoints of lines

I'm sure you could easily determine the midpoint of a horizontal or vertical segment. You could use the ruler postulate to determine the distance between the two points, divide it in half, and add to the first point. In so doing, you are basically finding the average of the two points.

But what if the line was slanted? We now have two distances; one along the *x*-axis, the other along the *y*-axis. Once again, Descartes to the rescue!

If you plot the line on the coordinate plane, you can see that you can project the line onto both axes. If you find the average of the distance along each axis, you can find the midpoint along that axis (add the average to the lower point along the axis). Project these points back onto the line, and you have the midpoint of the line!

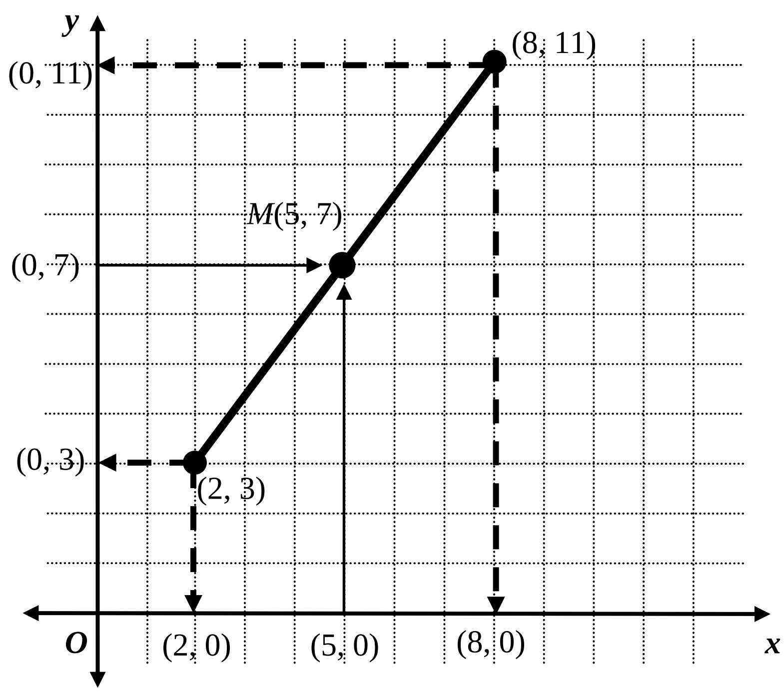
The average of the *y*-axis projection is 7:

$$\left(\frac{11+3}{2}\right)$$

The average of the *x*-axis projection is 5:

$$\left(\frac{8+2}{2}\right)$$

Projecting these two points back onto the line, we have a midpoint of (5, 7).



The midpoint formula

The midpoint *M* of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

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Example (pg. 46, #20)

H(13, 8) $x_1 = 13$ $y_1 = 8$
X(-6, -6) $x_2 = -6$ $y_2 = -6$

$$M(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$
$$= M(\frac{13 + (-6)}{2}, \frac{8 + (-6)}{2})$$
$$= M(\frac{13 - 6}{2}, \frac{8 - 6}{2})$$
$$= M(\frac{7}{2}, \frac{2}{2})$$
$$= M(3.5, 1)$$

Assign homework

p. 46 1-15 odd, 19-41 odd, 44, 48, 62, 66, 68